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Economics Department Working Paper Series | Economics

2021

What is the Impact of an Intensification of Labour on the Rate and Form of Exploitation?

Deepankar Basu
Department of Economics, UMass Amherst, dbasu@econs.umass.edu

Cameron Haas
Department of Economics, UMass Amherst, cdhaas@umass.edu

Athanasios Moraitis
Department of Economics, UMass Amherst, athanasiosmo@umass.edu

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Deepankar Basu* Cameron Haas† Athanasios Moraitis‡

February 8, 2021

Abstract

Does the intensification of labour increase the rate of exploitation? Does it produce absolute surplus value or relative surplus value? This paper develops a framework to answer these questions by incorporating intensity of labour in the widely-used linear model of production, both in its one and two department forms. We show, first, that an intensification of labour always leads to an increase in the rate of exploitation, and second, that the increase in the rate of exploitation takes the form of the production of absolute surplus value in all realistic situations. We also highlight, in the case of any model with more than one industry or sector, an interesting difference in short run and long run changes in the rate and form of surplus value.
JEL Codes: B51
Keywords: rate of exploitation; absolute surplus value; relative surplus value; linear model of production

1 Introduction

In Marxist economics, the rate of exploitation occupies a central position. It is a quantitative expression of a key feature of capitalism: the exploitation of labour by capital. The rate of exploitation is defined as the ratio of surplus labour and necessary labour. Converted to value-theoretic terms the rate of exploitation is equal to the ratio of surplus value generated in production and the value of labour power. Once the real wage bundle per

*Department of Economics, UMass, Amherst, MA 01003. `dbasu@econs.umass.edu`
†Department of Economics, UMass, Amherst, MA 01003. `cdhaas@umass.edu`
‡Department of Economics, UMass, Amherst, MA 01003. `athanasiosmo@umass.edu`

hour of labour is fixed, the rate of exploitation depends on the interaction of three variables: the length of the working day, the productivity of labour and the intensity of labour.

In Volume I of *Capital*, Marx discussed two different methods available to capitalism to increase the rate of exploitation: (a) the production of 'absolute surplus value'; and (b) the production of 'relative surplus value'.

> I call the surplus-value which is produced by the lengthening of the working day, *absolute surplus-value*. In contrast to this, I call that surplus-value which arises from the curtailment of the necessary labour-time, and from the corresponding alteration in the respective lengths of the two components of the working day, *relative surplus-value*. (Marx, 1992, pp. 432).

Among the three variables which interact to determine the rate of surplus value, the impact of changes in the length of the working day and the productivity of labour are easy to see. If there is an increase in the length of the working day, holding productivity and intensity of labour fixed, that is by definition the production of absolute surplus value. On the other hand, if the productivity of labour increases, holding the length of the working day and the intensity of labour fixed, then this leads to a production of relative surplus value. This is because an increase in the productivity of labour reduces the unit value of commodities, which, in turn, reduces the value of labour power because the real wage bundle is constant. Since the length of the working day is held constant, this is clearly an instance of the production of relative surplus value.

In contrast to this, the effect of changes in the intensity of labour on the production of absolute or relative surplus value is less obvious. This is because an increase in the intensity of labour has two distinct effects. On the one hand, it increases labour's capacity to process inputs into output (CPIO) per unit of time, which is akin to a pure increase in the productivity of labour. On the other hand, an intensification of labour also implies a larger expenditure of labour power per unit of time, which is like an increase in the length of the working day, albeit in an intensive sense (Marx, 1992, pp. 534), and hence increases labour's capacity to create value (CCV). The overall impact of the intensification of labour, being the result of these two effects, is not immediately obvious. The CPIO-effect tends to indicate towards the production of relative surplus value; the CCV-effect points in the direction of the production of absolute surplus value. This might be the reason behind the disagreement in the extant Marxist literature about the effect of the intensification of labour. While some scholars argue that an increase in the intensity of labour produces absolute surplus value (Foley, 1986; Catephores, 1989; Sekine, 1997; Joosung, 1999; Hudson, 2001; Heinrich, 2012), others claim that an intensification of labour leads to a production of relative surplus value (Philp et al., 2005; Mavroudeas and Ioannides, 2011). In fact, a close reading of Volume I of *Capital* shows that there is some ambiguity in Marx's analysis too.

On the one hand, there is ample scope to read Volume I and come away with the impression that in Marx's analysis, intensification of labour leads to the production of RSV and not ASV. To begin with, part 3 of Volume I of *Capital*, which is devoted to an analysis of the production of absolute surplus value (ASV), does not contain any serious discussion of intensification of labour. The main discussion of intensification of labour occurs in Chapter 15(c) in Volume I of *Capital*, which is located in Part 4 of Volume I, the part which is devoted to an analysis of the production of relative surplus value (RSV). Taken together, this suggests that in Marx's analysis, intensification of labour is relevant to the production of RSV and not ASV. When we delve into the discussion in Chapter 15(c), this supposition is further confirmed.

In Chapter 15(c), Marx discusses the intensification of labour in the context of development of machine production and large scale industry. Marx clearly distinguishing between production of ASV and the production of RSV, and identifies the prolonging of the working day as cause of the former. According to Marx, when legal restrictions prevent the elongation of the working day, capital turns to the production of RSV.

> As soon as the gradual upsurge of working-class revolt had compelled Parliament compulsorily to shorten the hours of labour, and to begin by imposing a normal working day on factories properly so called, i.e. from the moment that it was made impossible once and for all to increase the production of surplus-value by prolonging the working day, capital threw itself with all its might, and in full awareness of the situation, into the *production of relative surplus value*, by speeding up the development of the machine system (Marx, 1992, pp. 534, emphasis added).

It is only while commenting on how "capital threw itself with all its might ... into the production of relative surplus value", that Marx discuss effects of changes in the productivity and intensity of labour on the production of surplus value. Hence, it is difficult to not interpret Marx's discussion in Chapter 15(c) of Volume I as suggesting that intensification of labour leads to the production of RSV.

On the other hand, it is also clear that Marx distinguishes between the intensive and extensive margin of labour-time, that he thinks of intensification of labour as analogous to an extension of the working day - he does so explicitly in the same paragraph from which the above passage is quoted. This distinction implies that an intensification of labour might also produce ASV. But Marx does not work fully work out the implication of the distinction. He notes it and moves on. That, to our mind, is the source of the ambiguity in Volume I of *Capital* about the analysis of intensification of labour. Therefore, we think the disagreement in the later Marxist literature regarding the analysis of intensification of labour can be traced back, at least partly, to Marx's ambiguity.

Our aim in this paper is to provide a coherent framework to think about the value dimensions of the intensification of labour, to address the ambiguity in Marx's analysis and perhaps go some way in resolving existing disagreements about its effect with regard to the production of ASV and RSV. In concrete terms, our paper makes two important contributions. First, we offer a simple way of incorporating the intensity of labour in the linear model of production that is widely used in classical and Marxian economics. Our proposed method rests on clearly and quantitatively separating out labour's CPIO and CCV, so far as they are related to the intensity of labour. While most scholars have associated with intensification of labour what we have called labour's CCV, there is far less attention devoted to labour's CPIO. Second, using our model, we draw out the implications of an intensification of labour on the rate and form of surplus value in a variety of linear production models of the capitalist economy.

With regard to the rate of surplus value, we demonstrate the intuitive result that an intensification of labour always leads to an increase in the rate of surplus value. Hence, it is always in the interest of the capitalist class and always against the interest of the working class to increase the intensity of labour. We can therefore expect a pronounced class struggle over the intensity of labour in capitalist economies, much like the struggle in mid-19th century England over the length of the working day.

Our results about the form of surplus value are more complex because they depend on the relative magnitudes of labour's CPIO and CCV. We show that: (a) if labour's CPIO and CCV are impacted equally by the intensification of labour, then we have production of only absolute surplus value; (b) if, with the intensification of labour, labour's CPIO rises less than its CCV, even then we have production of only absolute surplus value; (c) if the intensification of labour is associated with labour's CPIO rising more than its CCV, then the rise in the rate of surplus value can be decomposed into the sum of two components, one associated with the production of absolute surplus value and the other associated with the production of relative surplus value.

The first case, where labour's CPIO and CCV are impacted equally by the intensification of labour, seems most common and would arise when intensification of labour does not exceed the normal limits of work and effort. The second case, where labour's CPIO rises less than its CCV by the intensification of labour, would occur when intensification of labour breaches normal working conditions and leads to exhaustion of workers. The third case, where labour's CPIO rises more than its CCV by the intensification of labour, seems unlikely to occur in any realistic scenario. Hence, we conclude that an intensification of labour will lead to the production of only absolute surplus value in all realistic scenarios.

Our analysis has important political implications. It has been common in the Marxist economics literature to associate the production of absolute surplus value with the early stages of capitalism. It is thought that once the length of the working day has been more or less determined by the struggle of the working class, the main method of raising the rate of surplus value is through the production of relative surplus value. Our analysis highlights that that is not the case. Even when the length of the working day has been fixed by working class struggle and State legislation, the capitalist class has the option of resorting to the production of absolute surplus value by intensifying the labour process. Just like 19th century England witnessed the epic struggles of the working class for regulating the length of the working day, it is equally important for the working class to fight for regulation of the intensity of labour today. Of course, the fact that the intensity of labour is much harder to measure makes the struggle for its regulation all the more difficult. This should not detract from its importance in contemporary class struggle.

The rest of the paper is organized as follows. In section 2, we use a one-commodity model of production with a linear technology, i.e. a corn model, to derive the main results of this paper. In section 3, we derive the same results in the simplest two department model with two commodities. Department I produces the single means of consumption and Department II produces the single means of production. The main points of our analysis can be fully understood in the context of the one-commodity model and the simple two department model, but for completeness, we also present all the results in a more general linear setting. In section 4, we present all the results in a general n-commodity model with a linear technology, and in section 5, we do so for a general two department model *à la* Morishima. The final section concludes the discussion with some thoughts about the broader implications of our research. Throughout this paper, we make the following simplifying assumptions: there is no fixed capital; labour is homogeneous; each commodity is produced with one technique of production (hence, we abstract from choice of technique); the output in each sector is only one commodity (hence, we abstract from joint products).

2 One-Commodity Linear Model

2.1 Technology and Value

Consider a capitalist economy which produces one commodity, called 'corn', and assume that it can be both consumed and invested. Production of corn requires both a nonlabour input, corn itself, and labour. Consider a benchmark case where it takes a units of corn and l units of labour to produce 1 unit of corn. In such a case, we represent the technology of production with the pair of numbers, (a, l).

Given this technological relationship, we can easily determine the value of a unit of corn. If we denote by λ the value of a unit of corn, we will have

$$\lambda = \lambda a + l,$$

where λa is the value transferred by the non-labour input and l is the value added by direct labour. Hence,

$$\lambda = \frac{l}{1-a} = \left[\frac{1}{(1/l)}\right] \times \left[\frac{1}{1-a}\right]. \tag{1}$$

For the technology represented by (a, l) to be feasible we need $0 < a < 1$, i.e. the amount of corn that is needed as input to produce 1 unit of corn is a positive fraction. The lower bound comes from the requirement that some corn be always used as inputs, and the upper bound comes from the requirement that more corn cannot be needed as input than what will be produced as output. We also assume that labour is essential to production, so that $l > 0$. These two assumptions applied to (1) ensure that the value of corn is positive.

2.2 Intensity of Labour

We would now like to define two important aspects of labout that will allow us to define the intensification of labour. By labour's capacity to process inputs into output (CPIO), we refer to the physical amount of inputs converted into output per unit of time. By labour's capacity to create value (CCV), we refer to the magnitude of expenditure of labour power per unit of time. With these two notions in place, we can now define the intensity of labour.[1]

Defintion 1. *Given the technology of production (a, l), we will say that there has been an intensification of labour if there are two real numbers $\mu_1 > 1$ and $\mu_2 > 1$, such that*

1. *l units of labour can convert $\mu_1 a$ units of corn as input into μ_1 units of corn as output; and*

2. *each hour of labour creates μ_2 units of value.*

In this definition, μ_1 captures labour's CPIO and μ_2 captures labour's CCV. The fact that $\mu_1 > 1$ implies that compared to the situation before intensification, each hour of labour now converts a larger physical magnitude of inputs into output. In a similar way, the assumption that $\mu_2 > 1$ captures the fact that when workers work with higher intensity, there is larger expenditure of labour power per unit of time, compared to the situation before intensification. Since expenditure of labour power creates value, intensification of labour increases labour CCV, and this is captured by $\mu_2 > 1$.[2]

To analyze the impact of intensification of labour on the value of corn, let λ' denote the value of 1 unit of corn after the increase in intensity of labour.[3] Then,

$$\lambda' \mu_1 = \lambda' \mu_1 a + \mu_2 l,$$

where, like before, the first term on the RHS, $\lambda' \mu_1 a$, captures value transferred by the corn input and the second term, $\mu_2 l$, captures value added by direct labour. Hence,

$$\lambda' = \frac{\mu_2 l}{\mu_1 (1 - a)}.$$

Using (1), we get

$$\frac{\lambda'}{\lambda} = \frac{\mu_2}{\mu_1}. \tag{2}$$

Here we see an interesting result. When there is an increase in the intensity of labour, the change in the value of corn depends on the relative magnitude

[1] This definition of intensity of labour is inspired by Steedman (1977, Chapter 6).

[2] These two aspects of the intensification of labour are discussed by Marx in the context of the development of machinery and large-scale industry (Marx, 1992, pp. 534).

[3] We abstract from issues of aggregate demand in this analysis.

of μ_1 and μ_2. If $\mu_1 = \mu_2$, then the value of corn remains unchanged; if $\mu_1 < \mu_2$, then the value of corn rises; if $\mu_1 > \mu_2$, then the value of corn falls.

2.3 Rate of Exploitation

Let the working day be T hours long and let the real wage bundle for a day's work be given by B units of corn, i.e. workers get a nominal wage for T hours of work which enables them to purchase B units of corn. Let us denote by b the real wage bundle per hour of work, i.e. $b = B/T$. In the benchmark situation, workers earn a real wage bundle b for every hour of work. Hence, surplus value produced per hour is given by $1 - \lambda b$. Since the value of labour power, i.e. variable capital, is defined to be λb, the rate of exploitation is given by

$$e = \frac{1 - \lambda b}{\lambda b}$$

so that

$$1 + e = \frac{1}{\lambda b}. \tag{3}$$

In the new situation, suppose there is an intensification of labour, where the latter is captured by μ_1 and μ_2 as specified in Definition 1. In this case, if λ' denotes the value of a unit of corn, then surplus value created per hour is given by $\mu_2 - \lambda' b$. The value of labour power, in this new situation is, given by $\lambda' b$. Hence, the rate of exploitation is given by

$$e' = \frac{\mu_2 - \lambda' b}{\lambda' b}$$

so that

$$1 + e' = \frac{\mu_2}{\lambda' b}. \tag{4}$$

We can now combine (3) and (4) to get

$$\frac{1 + e'}{1 + e} = \frac{\mu_2 \lambda b}{\lambda' b}.$$

Using (2), we get

$$\frac{1 + e'}{1 + e} = \mu_1. \tag{5}$$

Our first result can be stated as

Claim 1. *If the intensity of labour rises holding the real wage bundle per hour fixed, then the rate of exploitation rises.*

Proof. This can be seen immediately from (5) by noting that $\mu_1 > 1$ whenever there is an intensification of labour. $\square$

2.4 Form of Exploitation

If the intensity of labour rises, holding the hourly real wage bundle fixed, then the analysis of the form of surplus value is complicated because of the dependence of the result on the relative magnitudes of μ_1 and μ_2. In fact, we need to consider three cases. But before delving into these three cases, let us recall the key difference between the production of *absolute surplus value* and the production of *relative surplus value*. The key difference between these two ways to increase the rate of exploitation is whether there is what Marx calls "curtailment of the necessary labour-time", i.e. whether there is a decline in the value of labour power. If there is a decline in the value of labour power, then the increase in the rate of exploitation take the form of the production of relative surplus value; if the value of labour power does not decline, i.e. it either stays constant or rises, then an increase in the rate of exploitation takes the form of the production of absolute surplus value.

2.4.1 Case 1: $\mu_1 = \mu_2$

Consider the case in which $\mu_1 = \mu_2$. This condition means that an increase in the intensity of labour leads to an equal increase in it's CCV, captured by μ_2, as in its CPIO, captured by μ_1. From (2), we see that if $\mu_1 = \mu_2$, then $\lambda' = \lambda$. Thus, the value of each unit of corn remains unchanged. Since the hourly real wage bundle is fixed at b units of corn, the value of labour-power does not change. Moreover, value added in an hour's work has increased to μ_2 from 1, where $\mu_2 > 1$. Hence, this is clearly a case of the production of *absolute surplus value.*

2.4.2 Case 2: $\mu_1 < \mu_2$

Consider the case when when $\mu_1 < \mu_2$. This condition means that an increase in the intensity of labour leads to a lower increase in its CPIO, captured by μ_1, than in its CCV, captured by μ_2. From (2), we see if $\mu_1 < \mu_2$, then $\lambda' > \lambda$. Thus, the value of each unit of corn rises. Since the hourly real wage bundle is fixed at b units of corn, this leads to an increase in the value of labour-power. What happens to value added? Since value added in an hour's work is μ_2, where $\mu_2 > 1$, the value added increases in comparison to the original situation when it was 1. From Claim 1, know that $e' > e$. Is this production of absolute surplus value or relative surplus value? Recall that in this case, the value of labour-power has increased. That rules out the production of relative surplus value. We are left with the conclusion that here too we have a case of the production of *absolute surplus value.*

2.4.3 Case 3: $\mu_1 > \mu_2$

Consider the case when $\mu_1 > \mu_2$. This condition means that an increase in the intensity of labour leads to a higher increase in its CPIO, captured by μ_1, than in its CCV, captured by μ_2. From (2), we see that if $\mu_1 > \mu_2$, then $\lambda' < \lambda$. Hence the value of corn falls. Since the hourly real wage bundle is fixed at b units of corn, this leads to a fall in the value of labour-power. What happens to value added? Since value added by an hour's work is μ_2, where $\mu_2 > 1$, the value added per hour increases from its original magnitude of 1. Hence, surplus value increases. Is this production of absolute or relative surplus value?

Using (3) and (4), we see that

$$e' - e = \frac{\mu_2}{\lambda' b} - \frac{1}{\lambda b} = \frac{\mu_2 - 1}{\lambda' b} + \left(\frac{1}{\lambda' b} - \frac{1}{\lambda b} \right).$$

This shows that the change in the rate of surplus value comes from a combination of both absolute surplus value (the first term on the RHS) and relative surplus value (the second term on the RHS). The first term represents absolute surplus value because it captures an increase in labour's CCV, i.e. $\mu_2 > 1$, which increases the value added in an hour's work from 1 to μ_2. It is as if workers work μ_2 hours in place of every 1 hour *keeping intensity fixed*. That is why the first term can be associated with the production of *absolute surplus value*. The second term represents relative surplus value because it comes from a fall in the value of a unit of corn, i.e. $\lambda' < \lambda$. With a fall in the value of corn and the hourly real wage bundle fixed at b units of corn, there is a fall in the value of labour-power, i.e. $\lambda' b < \lambda b$. Hence, the second term indicates the production of *relative surplus value.*

2.5 Which Case is Relevant?

At some places in Volume I of *Capital*, Marx argued that an intensification of labour would keep the unit value of commodities unchanged.

> Increased intensity of labour means increased expenditure of labour in a given time. Hence a working day of more intense labour is embodied in more products than is one of less intense labour, the length of each working day being the same. Admittedly, an increase in the productivity of labour will also supply

> more products in a given working day. But in that case the value of each single product falls, for it costs less labour than before, whereas in the case mentioned here *that value remains unchanged*, because each article costs the same amount of labour as before (Marx, 1992, pp.660–661, emphasis added)

This understanding of the effect of an intensification of labour can be captured by Case 1. This is because, only in this case, i.e. when $\mu_1 = \mu_2$, does the value of corn remain unchanged after an intensification of labour. To our mind, this is the most common case, i.e. in ordinary circumstances, an intensification of labour is likely to increase its CPIO and CCV in equal magnitudes.

We might also entertain another possibility, i.e. when intensification of labour leads to a relatively lower increase in its CPIO than in its CCV. This can happen, for instance, when the intensification of labour leads to exhaustion of the workers beyond normal levels. In such a situation, their ability to handle inputs and covert them into output might be impaired. Thus, for every hour of time, workers might be expending a larger magnitude of labour power - reflecting an intensification of labour - but because of sheer exhaustion, their ability to convert input into output might have declined. Marx indirectly discusses this in some parts of Volume I of *Capital*.

> It is clear that if the value created by a day's labour increases from, say, 6 to 8 shillings, then the two parts into which this value is divided, namely the price of labour-power and surplus value, may both increase simultaneously, and either equally or unequally. They may both simultaneously increase from 3 shillings to 4. Here, the rise in the price of labour-power does not necessarily imply that it has risen above the value of labour-power. On the contrary, this rise in price may be accompanied by a fall below its value. This always occurs when the rise in the price of labour-power does not compensate for its more rapid deterioration (Marx, 1992, pp. 661).

Other Marxist scholars have highlighted this aspect more prominently, for instance Joosung (1999, pp. 186) and Mavroudeas and Ioannides (2011, pp. 431). In the framework developed in this paper, we can capture this situation by Case 2, i.e. where $\mu_1 < \mu_2$.

The third case, i.e. where $\mu_1 > \mu_2$, seems to us to be unrealistic and unlikely. We cannot conceive of any situation in which an intensification of labour leads to a larger increases in its CPIO than in its CCV. Hence, we think, while Case 3 is a logical possibility, it is unlikely to be of any interest when we are studying a real capitalist economy. It is difficult to conceive of situations when an intensification of labour leads to a relatively larger increase in labour's CPIO than in its CCV. This means that an intensification of labour, in all realistic scenarios, will lead *only* to the production of absolute surplus value.

3 Simple Two Department Model

So far the analysis has been conducted in the context of a one-commodity model. This is decidedly simple and unrealistic. Hence, we would now like to extend the analysis to more realistic and also more complicated setups. The first step in this direction is to consider an economy with two commodities, one a capital good and the other a wage good. We can think of this as a simplified version of Morishima's representation of the Marxian two department model (Morishima, 1973). The simplification we introduce at this point is that each department produces only one commodity. In section 5, we will relax this assumption and present results in a general two department model which produces n capital and $m-n$ wage goods.

Let us denote commodity 1 to be the capital good (produced in Department I) and commodity 2 as the wage good (produced in Department II). Note that if intensification of labour occurs *only* in Department II, then the analysis of the corn model in the previous section suffices to derive all results. Hence, here we consider the case where an intensification of labour occurs *only* in Department I. What can we say about its impact on the rate and form of surplus value?

3.1 Technology, Intensity and the Rate of Exploitation

Technology of production in the two departments are specified as given by (a_1, l_1) and (a_2, l_2): a_1 units of commodity 1 and l_1 hours of labour are required to produce 1 unit of commodity 1; and a_2 units of commodity 1 and l_2 hours of labour are needed to produce 1 unit of commodity 2. Let λ_1 and λ_2 denote the value of the capital and wage good. Then, we have

$$\lambda_1 = \frac{l_1}{1-a_1}, \tag{6}$$

and

$$\lambda_2 = a_2\lambda_1 + l_2 = \frac{l_1 a_2}{1-a_1} + l_2. \tag{7}$$

Let us denote by λ_1' and λ_2' the value of the capital and wage good, respectively, after an intensification of labour in the capital good sector. A little algebra shows that

$$\lambda_1' = \frac{\mu_2}{\mu_1}\lambda_1, \tag{8}$$

and

$$\lambda_2' = \frac{\mu_2}{\mu_1}\frac{l_1 a_2}{1-a_1} + l_2. \tag{9}$$

Let e_1 and e_1' denote the rate of exploitation in department I before and after intensification of labour. Much as in the corn model, we have

$$e_1 = \frac{1-b\lambda_2}{b\lambda_2},$$

and

$$e_1' = \frac{\mu_2 - b\lambda_2'}{b\lambda_2'},$$

so that

$$\frac{1+e_1'}{1+e_1} = \mu_2\frac{\lambda_2}{\lambda_2'}.$$

Claim 2. *If there is an intensification of labour in department I, the rate of exploitation in that department rises, i.e.* $e_1' > e_1$.

Proof. We have to consider three cases. Case 1. If $\mu_1 > \mu_2$, then a comparison of (7) and (9) shows that $\lambda_2' < \lambda_2$; so the rate of exploitation has increased. Case 2. If $\mu_1 = \mu_2$, then (6) to (9) shows that $\lambda_1' = \lambda_1$ and $\lambda_2' = \lambda_2$. Then $(1+e_1')/(1+e_1) = \mu_2 > 1$. Hence, $e_1' > e_1$. Case 3. If $\mu_2 > \mu_1$, then (7) and (9) shows that $\lambda_2' > \lambda_2$. But a little algebra also shows that

$$\lambda_2' = \mu_2\left[\frac{1}{\mu_1}\frac{l_1 a_2}{1-a_1} + \frac{l_2}{\mu_2}\right] < \mu_2\left[\frac{l_1 a_2}{1-a_1} + l_2\right] = \mu_2\lambda_2,$$

where the inequality follows because $\mu_1, \mu_2 > 1$. Hence, $1+e_1' > 1+e_1$, so that $e_1' > e_1$.

$\square$

3.2 Forms of exploitation

We have the same three cases to consider that we had encountered in the one-commodity model. If $\mu_1 = \mu_2$, then (7) and (9) shows that the value of the wage bundle stays the same. Since the rate of exploitation has increased, this is a form of absolute surplus value. If $\mu_1 < \mu_2$, then (7) and (9) shows that the value of labour power has increased. This rules out the production of relative surplus value. Since the rate of surplus value has increased, as demonstrated in Claim 2, this must be a case of the production of absolute surplus value. If $\mu_1 > \mu_2$, then (7) and (9) shows that the value of labour power has declined. To pin down the form of surplus value, note that

$$e_1' - e_1 = \frac{\mu_2}{\lambda_2' b} - \frac{1}{\lambda_2 b} = \frac{\mu_2 - 1}{\lambda_2' b} + \left(\frac{1}{\lambda_2' b} - \frac{1}{\lambda_2 b} \right).$$

Thus, in this case, we see a combination of the production of absolute surplus value (first term on the RHS) and the production of relative surplus value (second term on the RHS).

3.3 Short Run versus Long Run

The analyses of the two department model and the one commodity model have given us identical results thus far. But now we would like to highlight an important difference between the two models. As soon as we introduce more than one sectors (or industries) in the analysis, we face the following question: how does the intensity of labour compare across sectors? In any model with more than one sector, including the simple two department model being discussed in this section, we cannot have different intensities of labour across departments if, at the same time, we have the same real wage bundle and the same rate of exploitation across sectors. If intensities differ across sectors, then the rate of exploitation will also differ. For instance, the rate of exploitation in the two departments, after department I witnesses an intensification of labour, are different because

$$e_2' = \frac{1 - \lambda_2' b}{\lambda_2' b} \neq \frac{\mu_2 - \lambda_2' b}{\lambda_2' b} = e_1',$$

where the inequality follows because $\mu_2 > 1$.

This means that the analysis we have presented so far should be understood as a short run exercise. While we might have different intensities of labour across departments in the short run, class struggle and/or mobility of labour will ensure that every sector has the same intensity of labour in the long run, given that the hourly real wage bundle and the rate of exploitation are same across sectors.[4] There are many long run configurations that can

[4] It is a standard assumption in classical economics, coming down all the way from Adam Smith, that mobility of labour across sectors will equalize the rate of exploitation in the long run. If we give up this assumption in our analysis, then we can allow for different intensities of labour across sectors even in the long run.

emerge. To consider all the possibilities, let us consider two extreme cases.

The first extreme scenario will occur if the relative class power of labour is higher. In this scenario, labour will force the intensity to go down in department I. Thus, in the long run both departments will have the original intensity. The (common) rate of exploitation will fall back to the original level, e, after a temporary increase in department I. The second extreme scenario will occur if the relative class power of capital is higher. In this case, capital will force an increase in intensity in department II. If this happens, then both department will have the new intensity captured by (μ_1, μ_2). The rate of exploitation will increase in department II to become e'. The configuration that actually occurs might fall between these extreme cases, i.e. the outcome of class struggle leads to a new intensity of labour captured by (μ_1', μ_2') which is common to both departments and where $1 < \mu_1' < \mu_1$ and $1 < \mu_2' < \mu_2$.

The main results of this paper can all be understood in the context of the one-commodity model and the simple two department model. But for the sake of completeness, we will now extend them to general linear models. In the next section, we will present an extension of the argument of section 2 in the more general setting of a n-commodity economy; in the following section, we will extend the analysis of section 3 to the setting of a general 2 department economy, with n sectors in department I and $m - n$ sectors in department II.

4 General Linear Model

The economy under consideration produces n commodities using a linear technology; there is no fixed capital; labour is homogeneous; each commodity is produced with one technique of production (hence, we abstract from choice of technique); the output in each sector is only one commodity (hence, we abstract from joint products). Without loss of generality, we assume, for the analysis in this section, that intensification of labour occurs in sector 1.

4.1 Technology and Value

Consider an economy which produces n commodities using commodities and labour as inputs given by the technology $(\boldsymbol{A}, \boldsymbol{l})$, where

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \tag{10}$$

is the $n \times n$ matrix of input-output coefficients, and

$$\boldsymbol{l} = \begin{bmatrix} l_1 & l_2 & \cdots & l_n \end{bmatrix} \tag{11}$$

is the $1 \times n$ vector of direct labour inputs. In the above representation of technology, a_{ij} is the physical magnitude of the i-th commodity used to produce 1 unit of the j-th commodity, and l_j is the quantity of direct labour used to produce 1 unit of commodity j. Thus, given the technology $(\boldsymbol{A}, \boldsymbol{l})$, l_j units of direct labour works on $a_{1j}, a_{2j}, \ldots, a_{nj}$ units of the n commodities to produce 1 unit of commodity j.

Let $\boldsymbol{\lambda}$ denote the $1 \times n$ vector of values. Then, we have,

$$\boldsymbol{\lambda} = \boldsymbol{\lambda A} + \boldsymbol{l}.$$

We assume that $\boldsymbol{A}$ is productive, which ensures that $(\mathbf{I} - \boldsymbol{A})$ is invertible and each element of the inverse matrix is strictly positive. Hence, we get

$$\boldsymbol{\lambda} = \boldsymbol{l}(\mathbf{I} - \boldsymbol{A})^{-1}, \tag{12}$$

where λ is also strictly positive because $(\mathbf{I} - \boldsymbol{A})^{-1}$ and $\boldsymbol{l}$ are both strictly positive.

4.2 Intensity of Labour

Defintion 2. *Given technology $(\boldsymbol{A}, \boldsymbol{l})$, defined in (10) and (11), we say that there has been an intensification of labour if there exists two real numbers $\mu_1 > 1$ and $\mu_2 > 1$, such that*

1. *l_1 units of labour work with $\mu_1 a_{11}$, $\mu_1 a_{21}$, …, $\mu_1 a_{n1}$, units of commodity 1, 2, …, n, to produce μ_1 units of commodity 1; and*

2. *each hour of labour creates μ_2 units of value.*

A change in the intensity of labour has the following technological representation, $\left(\boldsymbol{A}', \boldsymbol{l}'\right)$, where

$$\boldsymbol{A}' = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \tag{13}$$

and

$$\boldsymbol{l}' = \begin{bmatrix} l_1\mu_2/\mu_1 & l_2 & \cdots & l_n \end{bmatrix} \tag{14}$$

Hence, the input-output matrix remains unchanged but there is a change in the first element of the direct labour input vector compared to the benchmark situation. Note that only the first element changes because intensification of labour occurs only in sector 1.

4.3 Rate of Exploitation

Let us suppose that the *real wage bundle per hour* is given by the nonnegative vector $\boldsymbol{b}$, where

$$\boldsymbol{b} = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix}$$

The rate of exploitation in the benchmark situation, i.e. before any change in the length of the working day, the productivity of labour or in the intensity of labour, is given by

$$e = \frac{1 - \boldsymbol{\lambda b}}{\boldsymbol{\lambda b}}. \tag{15}$$

After an intensification of labour, the rate of exploitation (in the sector which witnessed intensification of labour) is given by

$$e' = \frac{\mu_2 - \boldsymbol{\lambda}'\boldsymbol{b}}{\boldsymbol{\lambda}'\boldsymbol{b}}. \tag{16}$$

Hence,

$$\frac{1 + e'}{1 + e} = \frac{\mu_2 \boldsymbol{\lambda b}}{\boldsymbol{\lambda}'\boldsymbol{b}}. \tag{17}$$

Claim 3. *If there is an intensification of labour in any sector, then the rate of exploitation in that sector rises, i.e. $e' > e$.*

Proof. Let $\boldsymbol{c} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b} > 0$ where the strict positivity of the vector $\boldsymbol{c}$ comes from the strict positivity of $(\boldsymbol{I} - \boldsymbol{A})^{-1}$, because $\boldsymbol{A}$ is productive, and the nonnegativity of the wage bundle, $\boldsymbol{b}$. Then

$$\mu_2 \boldsymbol{\lambda b} = \mu_2 \boldsymbol{l} (\boldsymbol{I} - \boldsymbol{A})^{-1} \boldsymbol{b} = \mu_2 \boldsymbol{lc} = \mu_2 \sum_{i=1}^{n} l_i c_i, \tag{18}$$

and, using the definition of $\boldsymbol{l'}$ in (14),

$$\boldsymbol{\lambda' b} = (\mu_2/\mu_1) l_1 c_1 + \sum_{j=2}^{n} l_j c_j. \tag{19}$$

Hence,

$$\mu_2 \boldsymbol{\lambda b} - \boldsymbol{\lambda' b} = \mu_2 \left(1 - \frac{1}{\mu_1}\right) l_1 c_1 + [\mu_2 - 1] \sum_{j=2}^{n} l_j c_j.$$

If the intensity of labour increases, then we have $\mu_1 > 1$ and $\mu_2 > 1$. Hence, Hence, $\mu_2 \boldsymbol{\lambda b} - \boldsymbol{\lambda' b} = \mu_2 (1 - (1/\mu_1)) l_1 c_1 + (\mu_2 - 1) \sum_{j=2}^{n} l_j c_j > 0$. Hence, using the expression in (17), it follows that $e' > e$.

□

Claim 3 is the analogue of Claim 1 and Claim 2. It demonstrates that whenever there is an intensification of labour, the rate of exploitation of labour rises in the sector where intensification of labour has occurred.

4.4 Form of Exploitation

To prove the results about the form of exploitation, we note that, in the sector which witnessed an intensification of labour, the difference in the value of the wage bundle before and after the intensification of labour is given by

$$\boldsymbol{\lambda b} - \boldsymbol{\lambda' b} = \left(1 - \frac{\mu_2}{\mu_1}\right) l_1 c_1,$$

where we have used (18) with $\mu_2 = 1$, and (19) as it is. We have the following three cases to consider.

Case 1 : $\mu_2 = \mu_1$. In this case, $\boldsymbol{\lambda b} = \boldsymbol{\lambda' b}$, i.e. the value of the wage bundle does not change. This must mean that the increase in the rate of exploitation, which is established in Claim 3, arises in the form of absolute surplus value.

Case 2 : $\mu_1 < \mu_2$. In this case, $\boldsymbol{\lambda b} < \boldsymbol{\lambda' b}$, i.e. the value of the wage bundle increases. This rules out relative surplus value. Hence, it must mean that the increase in the rate of exploitation, which is established in Claim 3, arises in the form of absolute surplus value.

Case $3 : \mu_1 > \mu_2$. In this case, $\boldsymbol{\lambda b} > \boldsymbol{\lambda' b}$, i.e. the value of the wage bundle decreases. This must mean that the increase in the rate of exploitation arises, which is established in Claim 3, might be a combination of absolute surplus value and relative surplus value. To investigate the last case further, note that

$$e' - e = \frac{\mu_2}{\boldsymbol{\lambda' b}} - \frac{1}{\boldsymbol{\lambda b}} = \frac{\mu_2 - 1}{\boldsymbol{\lambda' b}} + \left(\frac{1}{\boldsymbol{\lambda' b}} - \frac{1}{\boldsymbol{\lambda b}} \right).$$

Hence, the increase in the rate of exploitation is the sum of a component that can be attributed to the production of absolute surplus value effect, $(\mu_2 - 1)/\boldsymbol{\lambda' b}$, and a second component that can be attributed to the production of relative surplus value, $(1/\boldsymbol{\lambda' b}) - (1/\boldsymbol{\lambda' b})$.

4.5 Short Run versus Long Run

The analysis of the general linear model faces the same issue that we had highlighted in the simple two department model: we cannot have different intensities of labour across sectors if, at the same time, we have the same real wage bundle and the same rate of exploitation across sectors. For instance, the rate of exploitation in sector 1, where an intensification of labour first occurred, would be different from the rate of exploitation in any other sector in the economy because, for any $j \neq 1$,

$$e'_j = \frac{1 - \boldsymbol{\lambda' b}}{\boldsymbol{\lambda' b}} \neq \frac{\mu_2 - \boldsymbol{\lambda' b}}{\boldsymbol{\lambda' b}} = e'_1.$$

Hence, our analysis should be understood as a short run exercise. In the long run, class struggle and/or mobility of labour will ensure that every sector has the same intensity of labour, the same real wage bundle and hence the same rate of exploitation. As we have pointed out in the context of the simple two department model, there are many possible long run configurations bracketed by two extreme cases. In the first extreme case, the intensity of labour returns to the original level in the long run, thus ensuring that the intensity is same across all sectors. In this case, the rate of exploitation falls back to its original level. The other extreme case would occur is the intensity of labour were increased to the level that obtains in sector 1 for all sectors indexed by $j \neq 1$. In this case, the rate of exploitation rises in all sectors of the economy. The actual outcome could lie between these extreme scenarios.

5 General Two Department Model

5.1 Technology and Value

Consider a general two department Marxian model, as proposed by Morishima (1973), where the economy produces m commodities.[5] Of these m commodities, the first n are capital goods (means of production), and the remaining $m-n$ are wage goods (means of consumption). All capital goods are produced in Department I and are used in the production of other commodities; on the other hand, wage goods are produced in Department II and enter into the wage bundle $\boldsymbol{b}$. We write the technology of production in department I and II as $(\boldsymbol{A_I}, \boldsymbol{L_I})$ and $(\boldsymbol{A_{II}}, \boldsymbol{L_{II}})$, respectively, where

$$\boldsymbol{A_I} = \begin{bmatrix} a_{11} & ... & a_{1n} \\ . & & . \\ . & & . \\ . & & . \\ a_{n1} & ... & a_{nn} \end{bmatrix}, \boldsymbol{A_{II}} = \begin{bmatrix} a_{1(n+1)} & ... & a_{1m} \\ . & & . \\ . & & . \\ . & & . \\ a_{n(n+1)} & ... & a_{nm} \end{bmatrix}$$

where $\boldsymbol{A_I}$ is an $n \times n$ and $\boldsymbol{A_{II}}$ is an $n \times (m-n)$ input-output matrix, and the corresponding labour input vectors are

$$\boldsymbol{L_I} = (l_1, ..., l_n), \boldsymbol{L_{II}} = (l_{n+1}, ..., l_m),$$

which give the number of labour hours required to produce a unit of each commodity. Without loss of generality, we assume that if there is a change in intensity of labour it occurs either in sector 1 (the first industry in department I) or sector $n+1$ (the first industry in department II). In either case, the material input-output matrices, $\boldsymbol{A_I}$ and $\boldsymbol{A_{II}}$ remain unchanged, but the labour input vectors change to $\boldsymbol{L'_I} = \left(\frac{\mu_2}{\mu_1} l_1, ..., l_n\right)$ or $\boldsymbol{L'_{II}} = \left(\frac{\mu_2}{\mu_1} l_{n+1}, ..., l_m\right)$.

The values of the m commodities will be given by the following system of equations

$$\boldsymbol{\Lambda_I} = \boldsymbol{\Lambda_I} \boldsymbol{A_I} + \boldsymbol{L_I}$$
$$\boldsymbol{\Lambda_{II}} = \boldsymbol{\Lambda_I} \boldsymbol{A_{II}} + \boldsymbol{L_{II}}$$

where $\boldsymbol{\Lambda_I}$ and $\boldsymbol{\Lambda_{II}}$ denote the $1 \times n$ and $1 \times (m-n)$ vector of values in the two departments, respectively. We will use $\boldsymbol{\Lambda'_I}$ and $\boldsymbol{\Lambda'_{II}}$ to denote the value

[5]Recall that, in this paper, we abstract from fixed capital, joint products, choice of technique and heterogeneous labour.

vectors after an intensification of labour in department I and $\boldsymbol{\Lambda''_I}$ and $\boldsymbol{\Lambda''_{II}}$ to denote the value vectors after an intensification of labour in department II.

5.2 Rate of Exploitation

Let e and e' denote rates of exploitation before and after an intensification of labour in sector 1 in department I, respectively. Then,

$$e = \frac{1 - \boldsymbol{\Lambda_{II} b}}{\boldsymbol{\Lambda_{II} b}}, e' = \frac{\mu_2 - \boldsymbol{\Lambda'_{II} b}}{\boldsymbol{\Lambda'_{II} b}}.$$

We have further that

$$\frac{1+e'}{1+e} = \frac{\mu_2 \boldsymbol{\Lambda_{II} b}}{\boldsymbol{\Lambda'_{II} b}}$$

Claim 4. *If there is an intensification of labour in sector 1 in department I, then the rate of exploitation rises in that sector, i.e. $e' > e$.*

Proof. Assuming that the technology of production in both departments is productive, we have the following value system

$$\begin{aligned} \boldsymbol{\Lambda_I} &= \boldsymbol{L_I}(\boldsymbol{I} - \boldsymbol{A_I})^{-1} \\ \boldsymbol{\Lambda_{II}} &= \boldsymbol{L_I}(\boldsymbol{I} - \boldsymbol{A_I})^{-1}\boldsymbol{A_{II}} + \boldsymbol{L_{II}}. \end{aligned}$$

Let $\boldsymbol{c} = (\boldsymbol{I} - \boldsymbol{A_I})^{-1}\boldsymbol{A_{II} b} > 0$, where positivity of $\boldsymbol{c}$ is guaranteed by the productivity of $\boldsymbol{A_I}$ and the non-negativity of $\boldsymbol{b}$. Then

$$\mu_2 \boldsymbol{\Lambda_{II} b} = \mu_2 \boldsymbol{L_I c} + \mu_2 \boldsymbol{L_{II} b} = \mu_2 \sum_{i=1}^{n} l_i c_i + \mu_2 \boldsymbol{L_{II} b}$$

By definition of $\boldsymbol{L'_I}$, we have

$$\boldsymbol{\Lambda'_{II} b} = \frac{\mu_2}{\mu_1} l_1 c_1 + \sum_{i=2}^{n} l_i c_i + \boldsymbol{L_{II} b}$$

which gives

$$\begin{aligned} &\mu_2 \boldsymbol{\Lambda_{II} b} - \boldsymbol{\Lambda'_{II} b} \\ &= \mu_2 \left(1 - \frac{1}{\mu_1}\right) l_1 c_1 + (\mu_2 - 1) \sum_{i=2}^{n} l_i c_i + (\mu_2 - 1)\boldsymbol{L_{II} b} > 0 \end{aligned}$$

as every term is strictly positive. $\square$

We can analogously define the rate of exploitation after an intensification of labour in department II as

$$e'' = \frac{\mu_2 - \boldsymbol{\Lambda''_{II} b}}{\boldsymbol{\Lambda''_{II} b}},$$

and establish a similar result.

Claim 5. *If there is an intensification of labour in department II, then the rate of exploitation rises, i.e. $e'' > e$.*

Proof. We have

$$\frac{1+e''}{1+e} = \frac{\mu_2 \boldsymbol{\Lambda_{II} b}}{\boldsymbol{\Lambda''_{II} b}}.$$

As in the preceding proof, let $\boldsymbol{c} = (\boldsymbol{I} - \boldsymbol{A_I})^{-1}\boldsymbol{b}$. We have

$$\mu_2 \boldsymbol{\Lambda_{II} b} = \mu_2 \boldsymbol{L_I c} + \mu_2 \boldsymbol{L_{II} b}$$

and

$$\boldsymbol{\Lambda''_{II} b} = \boldsymbol{L_I c} + \boldsymbol{L''_{II} b}$$

Therefore

$$\begin{aligned} &\mu_2 \boldsymbol{\Lambda_{II} b} - \boldsymbol{\Lambda''_{II} b} \\ &= (\mu_2 - 1)\boldsymbol{L_I c} + (\mu_2 - 1)\sum_{i=2}^{n-m} l_{n+i} b_i + (\mu_2 - \frac{\mu_2}{\mu_1}) l_{n+1} b_1 > 0 \end{aligned}$$

because $\mu_1, \mu_2 > 1$, which makes every term positive. □

Thus, whether the intensification of labour occurs in the production of capital goods or wage goods, the rate of exploitation increases in the sector where there is an intensification of labour.

5.3 Form of exploitation

For the sector in department I that witnessed the intensification of labour, we can rewrite the change in the rate of exploitation before and after the intensification of labour, as

$$e' - e = \frac{\mu_2}{\boldsymbol{\Lambda'_{II} b}} - \frac{1}{\boldsymbol{\Lambda_{II} b}} = \frac{\mu_2 - 1}{\boldsymbol{\Lambda'_{II} b}} + \left(\frac{1}{\boldsymbol{\Lambda'_{II} b}} - \frac{1}{\boldsymbol{\Lambda_{II} b}}\right).$$

Based on our work to establish $e' > e$, we have, as in the general linear case,

$$\mathbf{\Lambda}'_{\boldsymbol{II}}\boldsymbol{b} - \mathbf{\Lambda}_{\boldsymbol{II}}\boldsymbol{b} = \left(1 - \frac{\mu_2}{\mu_1}\right) l_1 c_1,$$

so the classification of form of exploitation based on μ_1, μ_2, is unchanged from the general linear case when an intensification of labour occurs in the capital goods department.

In a similar manner, when there is an intensification of labour in department II, the change in the rate of exploitation in the particular sector that witnessed the intensification of labour, is given as

$$e'' - e = \frac{\mu_2}{\mathbf{\Lambda}''_{\boldsymbol{II}}\boldsymbol{b}} - \frac{1}{\mathbf{\Lambda}_{\boldsymbol{II}}\boldsymbol{b}} = \frac{\mu_2 - 1}{\mathbf{\Lambda}''_{\boldsymbol{II}}\boldsymbol{b}} + \left(\frac{1}{\mathbf{\Lambda}''_{\boldsymbol{II}}\boldsymbol{b}} - \frac{1}{\mathbf{\Lambda}_{\boldsymbol{II}}\boldsymbol{b}}\right).$$

Based on our work to establish that $e'' > e$, we also have

$$\mathbf{\Lambda}''_{\boldsymbol{II}}\boldsymbol{b} - \mathbf{\Lambda}_{\boldsymbol{II}}\boldsymbol{b} = \left(1 - \frac{\mu_2}{\mu_1}\right) l_1 c_1,$$

so the same analysis applies when the intensification happens in department II.

5.4 Short Run versus Long Run

It is easy to see that the analysis of the general two department model faces the same issue that we had highlighted in the simple two department model, viz., that we cannot have different intensities of labour across sectors if, at the same time, we have the same real wage bundle and the same rate of exploitation across sectors. The analysis of this issue in the general linear model captures the key issues and we can only emphasize the same point we made in the previous section. Our analysis of the general two department model presented in this section should be understood as a short run exercise. In the long run, class struggle and/or mobility of labour will ensure that every sector has the same intensity of labour, the same real wage bundle and hence the same rate of exploitation. As we have pointed out in the context of the simple two department model and in the context of the general linear one department model, there are many possible long run configurations bracketed by two extreme cases. In the first extreme case, the intensity of labour returns to the original level in the long run, thus ensuring that the intensity is same across all sectors. In this case, the rate of exploitation falls back to its original level. The other extreme case would occur if the intensity of labour were increased to the level that obtains in sector 1, if the intensification occurs in department I or in sector $n+1$ if the intensification occurs in department II, for all other sectors. In this case, the rate of exploitation rises in all sectors of the economy. Of course, as we have pointed out above, the actual long run outcome could lie between these extreme scenarios.

6 Conclusion

As a system of social production built on exploitation, capitalism is driven by the need to continuously generate, realize and accumulate surplus value, the ultimate source of which is the unpaid labour of the working class. Competitive pressures in capitalism enforce the systemic need to keep increasing the rate of exploitation. Capitalism has two broad methods to increase the rate of exploitation: the production of absolute surplus value, and the production of relative surplus value.

The production of absolute surplus value can happen when the length of the working day increases, holding the productivity and intensity of labour fixed. By most historical accounts, the production of absolute surplus value was the main way of increasing the rate of exploitation during the early phases of capitalism. What happens when the struggle of the working class manages to force the State to regulate the length of the working day? Does capitalism then abandon the production of surplus value and focus solely on the production of relative surplus value? Does it revolutionize production and increase the productivity of labour - because that is the way to produce relative surplus value?

When class struggle by the workers has forced the State to put a limit on the length of the working day, the capitalist system does not lose its ability to produce absolute surplus value. While production of relative surplus value, and therefore growth of labour productivity, becomes important, simultaneously the system often uses intensification of labour - which is another reliable way to produce absolute surplus value.

In this paper, we have developed a simple way to incorporate intensification of labour into the widely-used linear model of production. Using this model we have demonstrated that an intensification of labour always increases the rate of exploitation. Hence, it is in the interest of the capitalist class, and against the interest of the working class, to increase the intensity of labour. We have also demonstrated, in various versions of the linear model of production, that an intensification of labour can, in most realistic situations, only produce absolute surplus value.

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